ANALYSIS OF COMPLEX PASSIVE (M)MIC COMPONENTS USING THE FINITE DIFFERENCE TIME-DOMAIN APPROACH

M. Rittweger and Ingo Wolff, Fellow, IEEE *

Abstract

The analysis of geometrical complex microstrip components including real three-dimensional structures using the finite difference time-domain (FDTD) method is presented. Frequency dependent characteristics of these components are received via Fourier transforms of the calculated time-domain results. Comparisons with accurate measurements are used to discuss the advantages and disadvantages of the applied method as well as its typical errors.

Introduction

Computer aided design of monolithic integrated microwave circuits requires the accurate description of interacting between single circuit elements caused by coupling effects. Numerical simulations of wave propagation phenomena can be used to develop fast models for circuit design as well as for testing the finished design before realization. Increasing computer capabilities improve the applicability of the known simulation techniques and additionally excite new investigations to receive better performance. Normally simulations of microwave components are performed in the frequency-domain because the time dependent wave equation reduces to the Helmholtz equation for harmonic fields. Alternatively [1,2] dealed with the time-domain analysis, which for the first time was used to calculate frequency dependent characteristics of microstrip discontinuities, instead of exclusively using time-domain results for illustrating wave propagation.

The FDTD-Method

The FDTD-method is a numerical method for the solution of electromagnetic field problems which has a large numerical but a low analytical expense. Despite the large numerical expense it is believed to be one of the most efficient techniques, because basically it only stores the field distribution at one moment in memory instead of working with a large system matrix relating several unknowns. The field solution for each other time then is determined by Maxwell's equations and is calculated using a time-stepping procedure based on the finite difference method. The used leapfrog algorithm fits very well on modern computer architectures, so that the data required to describe a three-dimensional field distribution can be handled in a reasonable time. Therefore it can efficiently be implemented on vector or parallel computers as well. Sufficiently accurate results can be received by using a single precision floating point expression requiring only four bytes as it has been shown in [7].

The closely related time-domain method of lines (TDML) [9] e.g. reduces the three-dimensional field problem of a *planar* structure into a two dimensional one, but it requires the evaluation of the eigenvalues of an up to (NM)$\times$(NM)-matrix, depending on the complexity of the metallization structure, where N and M are the numbers of discretizing lines in the two directions parallel to the metallization-plane. It is evident that the memory requirements of this full matrix can hardly surpass that of the 6NML-cube which represents the complete electromagnetic field of the discretized space in the case of the FDTD-method. L is the number of space-steps used for the discretization in the direction perpendicular to the substrate. Furthermore, the TDML requires magnetic or electric walls at the boundaries of the metallization-plane because the difference operators incorporate these boundary conditions. Some investigators of the FDTD-method also use exclusively such perfect reflecting walls. In consequence, the lines connected to the calculated structure have to be very long, to allow the separation of the scattered pulses if a transient analysis is prefered. Alternatively a particular solution is calculated in a resonant structure. In the first case the computational expense is blown up, in the second case it is questionable at all, whether the main advantage of the time-domain analysis is used.

The transient analysis delivers the broadband frequency response in one single computation run. A simulation of absorbing boundaries is necessary to match the ports using a finite discretized space and therefore a finite computer memory.

Boundary Truncation

In the case of the solution of an eigenvalue problem in a resonant structure the calculation is stopped after a time, which is sufficiently long to obtain approximately steady state solutions in the frequency-domain. The solution of wave propagation on the transmission line with its eigenfrequency is not touched from any transverse resonant phenomena, because they have eigenfrequencies of their own. In the other case of a transient analysis reflections at so called hard truncation conditions disturbe the results transformed in the frequency-domain considerably.

* The authors are with the Department of Electrical Engineering and Sonderforschungsbereich 254, Duisburg University, Bismarckstr. 69, D-4100 Duisburg, FRG.

This is due to the superposition of the investigated pulse on the transmission line and the mentioned reflections. This is the second reason that recommends the usage of artificial absorbing boundary conditions. Furthermore radiation is considered applying the FDTD-method in this way.

The available absorbing boundary conditions could be greatly improved using the super-absorption method [8], which for the first time was used for analyzing microstrip circuits in [1]. The application of the super-absorption method can not improve stability of absorbing boundaries which is important for calculations requiring a high number of time steps. The algorithm applied in [1] is not stable and so discretization noise can cause high frequency oscillations after a long calculation time. In this case the application of n-th order absorbing boundary algorithms [8] is more succesfull.

Nonequidistant Discretization

Equidistant discretization guarantees second order accuracy for the leapfrog algorithm. A rectangular shaped mesh can be used instead of a square shaped one without violating the available accuracy. This makes it possible to choose arbitrary ratios e.g. for the width of a microstrip and the substrate height whereas it has to be considered that the highest field variations can be resolved with the used grid. Numerical dispersion for wave propagation in directions with different spatial discretization also is different which means that an anisotropic behaviour must be considered if the ratios of the space-element borders strongly differ from unity. Nonequidistant discretization for finite difference schemes generally requires keeping the distances in the transverse directions constant i.e. it does not allow local mesh refinement without using interpolation. Due to this fact efficiency increases not as well as theoretically conceivable. In [3] this technique has been proposed, but not in the case of a transient analysis but in solving the eigenvalue problem of a resonant structure. The reason, why nonequidistant discretization in the cross section does not touch accuracy considerably in solving the eigenvalue problem, is precisely the same as described above. Nonequidistant discretization in the direction of wave propagation decreases accuracy rapidly. Reflections occurring in transition areas cause the same problems as that of reflecting boundaries. So for the transient analysis nonequidistant discretization must be handled very carefully. If geometrical dimensions of a metallization structure require the usage of nonequidistant discretization for exact description, the transition between neighboured space elements should be selected as smooth as possible. This is due to the central difference nature of the used algorithm. Nonequidistant discretization generally forbids forming precisely central differences, so the order of the algorithm decreases down to one. For the time-domain analysis this causes an indistinctness in the contour of the component i.e. makes relative convergence worse.

If nonequidistant discretization is used to expand the computation domain i.e. to increase the distance to the boundaries, it must be considered that the cutoff frequency

$$f_c = \frac{1}{\pi \Delta t} \arcsin\left(\frac{\Delta t}{(\varepsilon\mu)^{0.5}\, \Delta h} \right) \tag{1}$$

of the largest space-step Δh is clearly lower than the the highest spectral component of the scattered pulse. Otherwise the rough discretized area is not transparent for wave propagation. The most precise way to increase the size of space elements is to choose dimensions so, that always central differences can be used by modifying the leapfrog scheme. This modification is simply to use next neighboured field nodes instead of direct neighboured ones. Corresponding to the description shown in Fig. 1 the leapfrog algorithm for one-dimensional wave propagation can be formulated as follows:

$$H_i^{n+1/2} = H_i^{n-1/2} + \frac{\Delta t}{\mu\, \Delta h_i^e} \left(E_{i+1}^n - E_i^n \right), \tag{2}$$

$$E_i^{n+1} = E_i^n + \frac{\Delta t}{\varepsilon\, \Delta h_i^h} \left(H_i^{n+1/2} - H_{i-1}^{n+1/2} \right). \tag{3}$$

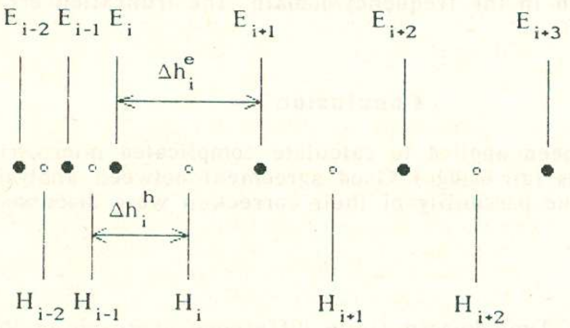

Fig. 1: Noneqidistant discretization for one-dimensional wave propagation.

To determine the time derivation of E_i, which is assumed to be an electric field node between space elements with different sizes (ratio 1/3 in this case), equation (3) changes to

$$E_i^{n+1} = E_i^{n} + \frac{\Delta t}{\varepsilon(\Delta h_i^h + \Delta h_{i-1}^h)} \left(H_i^{n+1/2} - H_{i-2}^{n+1/2} \right) \qquad (4)$$

in order to use a central difference to approximate the spatial derivation of the corresponding H-field.

Examples

The FDTD-method offers the possibility of a flexible problem formulation with an analytical expense which is much lower than e.g. that of spectral domain analysis techniques. Arbitrarily shaped planar line discontinuities and even three-dimensional elements as e.g. air-bridges or dielectric resonators can easily be approximated in a rectangular mesh. Even very rough mesh sizes lead to acceptable accuracy of the results. As an example for this thesis a meander line [5], shown in Fig. 3, has been discretized with only four spatial steps over the width of the microstrip line and the substrate height. The comparison of the calculated S-parameters (Fig. 4) with accurate measurements [4] (using the time-domain option of the network analyzer) shows that the amplitude and phase response of this complicated structure is described very accurately. In Figs. 2 and 3 additional information of the electric field inside the structure is given by the time dependent E_z field component over the substrate area and the time dependent voltage at port 1 (left port) and port 2 (right port) of the structure.

A typical effect caused from the discretization error of the applied method can be seen in the results (Fig. 6) of the analysis of a rectangular inductor: this is a frequency offset increasing for high frequencies. Nevertheless the analysis, considering all coupling effects of the lines and between the discontinuities including the influence of the air-bridge, shows good agreement in the comparison with measurements. Even the phase response which has been a critical aspect in nearly all other calculation methods is predicted with a high accuracy by the FDTD-method.

Errors

The results make believe that the assumptions made for the application of the FDTD-method, i.e. perfect conductors and no material losses, enable simulation of wave propagation in the presented (M)MIC-components with acceptable accuracy. Radiation effects are dominant compared to metallization losses, as it can be concluded from the fact that the attenuation of the components is described accurately. Errors do not result in an amplitude error but in a frequency offset mentioned above. This is essentially caused by the phenomena described below due to the discretized nature of the FDTD-method, which are very similar to those of the transmission line matrix (TLM) method [6].

In the case of equidistant discretization the used leapfrog algorithm is able to simulate wave propagation without increasing or decreasing amplitude up to the cutoff frequency (1) which corresponds to a wavelength of a few space elements depending on the used difference factors. Only a small dispersion which means a weakly frequency dependent phase-velocity occurs. This dispersion is a little bit higher for wave propagation in main axis direction than for arbitrary directions. The frequency shift for the calculated examples is less than 0.3 percent. An error correction after the simulation is possible by correcting the frequencies after Fourier transforms.

High non-uniform fields e.g. at the edge of a metallization can not be resolved from the used grid. This so called coarseness error is reported in [3] with the result, that the metallized contour is assumed to be too small, if it lies exactly on the grid nodes. This means, that the FDTD-method applied straight forward, as it has been done with the examples, calculates structures with metallization dimensions larger than those of the measured structures. This would declare why the resonant frequencies are calculated too small. This phenomenon has to be considered when the structure is discretized.

Reflections at the absorbing boundaries also cause errors as well as the truncation of the pulses before the transformation in the frequency-domain. The truncation error may cause ripples in the frequency response.

Conclusion

The FDTD-method has been applied to calculate complicated microstrip components including three-dimensional elements (air-bridge). Good agreement between analysis and measurements could be obtained. Errors and the possibility of their correction were discussed.

References

[1] X. Zhang, and K. K. Mei, "Time-domain finite difference approach to the calculation of the frequency-dependent characteristics of microstrip discontinuities," *IEEE Trans. Microwave Theory and Tech.*, vol MTT-36, pp. 1775-1787, Dec. 1988.

[2] X. Zhang, J. Fang, and K. K. Mei, "Calculations of the dispersive characteristics of microstrips by the time-domain finite difference method," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-36, pp. 263-267, Feb. 1988.
[3] C. J. Railton, and J. P. McGeehan, "Analysis of microstrip discontinuities using the finite difference time domain technique," IEEE MTT-S Digest, pp. 1009-1012, June 1989.
[4] G. Gronau, and I. Wolff, "A simple broad-band device de-embedding method using an automatic network analyzer with time-domain option," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-37, pp. 479-483, Jan. 1989.
[5] W. Wertgen, "Elektrodynamische Analyse geometrisch komplexer (M)MIC- Strukturen mit effizienten numerischen Strategien," Ph.D. Thesis, Duisburg University, FRG, 1989.
[6] W. J. R. Hoefer, "The transmission-line matrix method - theory and application," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-33, pp. 882-893, Oct. 1985.
[7] W. K. Gwarek, "Analysis of arbitrarily shaped two-dimensional microwave circuits by finite-difference time-domain method," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-36, pp. 738-744, April 1988.
[8] K. K. Mei, and J. Fang, "Super-absorption: a method to improve local absorbing boundary conditions," submitted for publication in *J. Comput. Physics*.
[9] S. Nam, H. Ling, and T. Itoh, "Time-domain method of lines applied to the uniform microstrip line and its step discontinuity," IEEE MTT-S Digest, pp. 997-1000, June 1989.

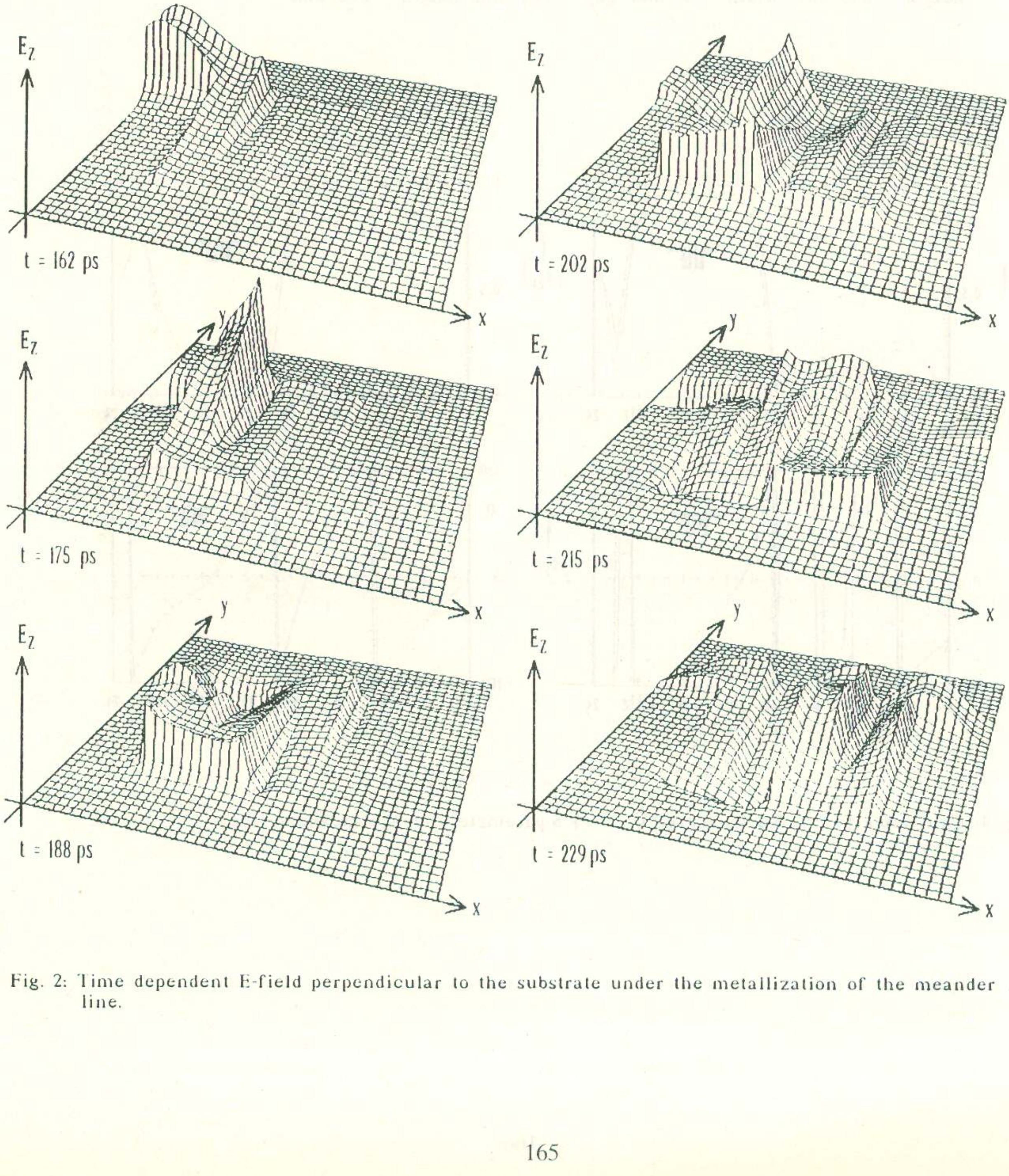

Fig. 2: Time dependent E-field perpendicular to the substrate under the metallization of the meander line.

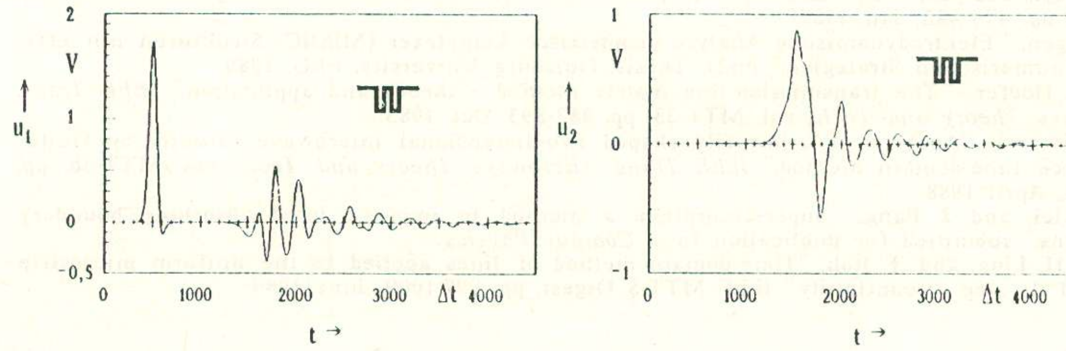

Fig. 3: Voltage history at port 1 (left port) and port 2 (right port) of the meander line. Al_2O_3, ε_r = 9.8, height = .635 mm, width = .61 mm, gap = .305 mm, length = 3.05 mm.

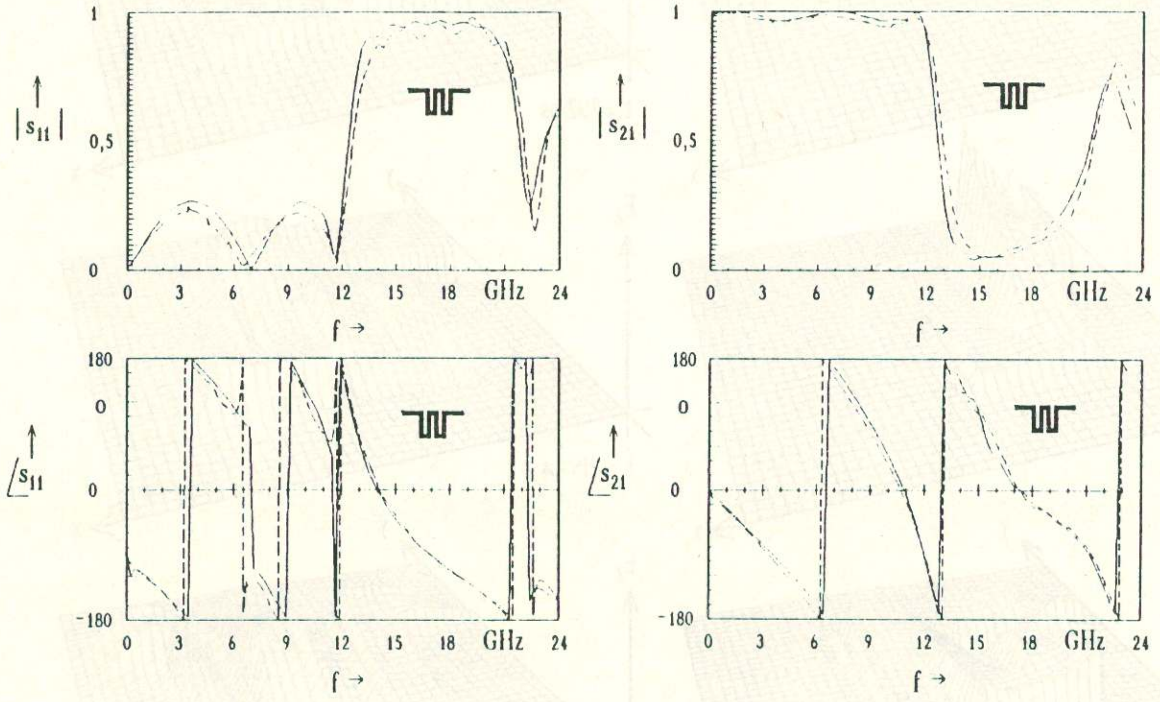

Fig. 4: Calculated (——) and measured (– – –) S-parameters of the meander line.

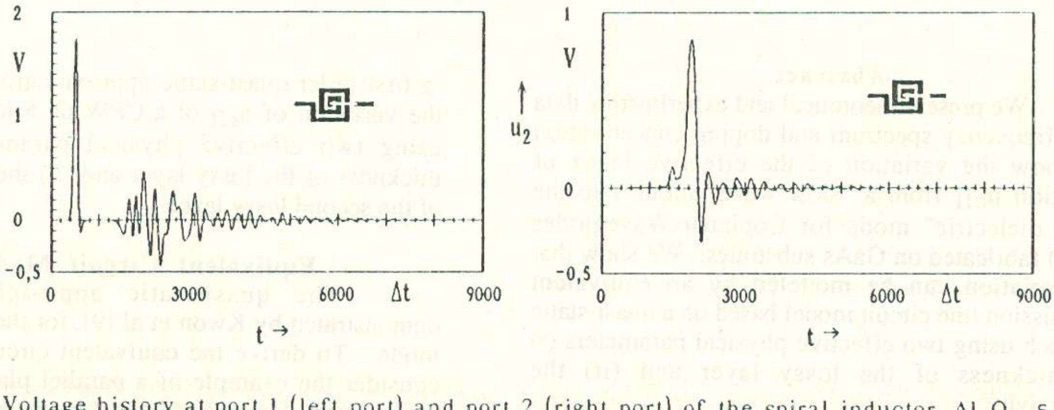

Fig. 5: Voltage history at port 1 (left port) and port 2 (right port) of the spiral inductor. Al_2O_3. ε_r = 9.8. height = .635 mm. width = .625 mm. gap = .3125 mm. air-bridge diameter = .3125 mm.

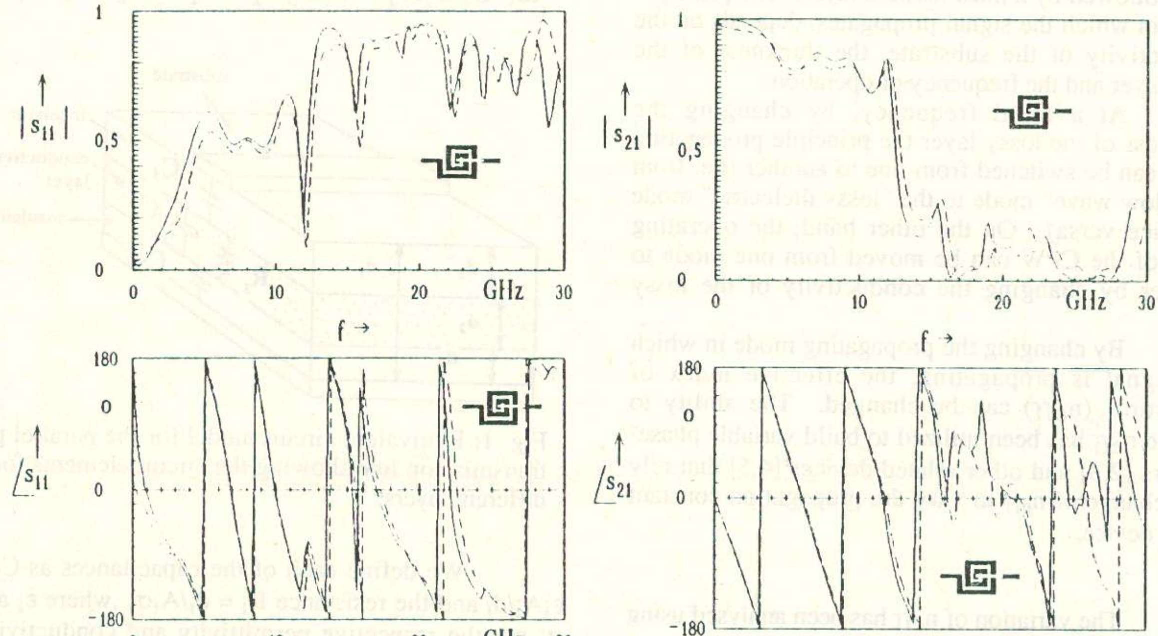

Fig. 6: Calculated (——) and measured (— — —) S-parameters of the spiral inductor.